

Modified geodetic brane cosmology

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Abstract. We explore the cosmological implications provided by the geodetic brane gravity action corrected by an extrinsic curvature brane term, describing a codimension-1 brane embedded in a 5D fixed Minkowski spacetime. In the geodetic brane gravity action we accommodate the correction term through a linear term in the extrinsic curvature swept out by the brane. We study the resulting geodetic-type equation of motion. Within a Friedmann-Robertson-Walker metric, we obtain a generalized Friedmann equation describing the associated cosmological evolution. We observe that, when the radiation-like energy contribution from the extra dimension is vanishing, this effective model leads to a self-(non-self)-accelerated expansion of the brane-like universe in dependence on the nature of the concomitant parameter β associated with the correction, which resembles an analogous behaviour in the DGP brane cosmology. Several possibilities in the description for the cosmic evolution of this model are embodied and characterized by the involved density parameters related in turn to the cosmological constant, the geometry characterizing the model, the introduced β parameter as well as the dark like-energy and the matter content on the brane.

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1. Introduction

A lot of attention has been paid to explain the current accelerated expansion of the Universe. As is well known, General Relativity (GR) cannot explain this fact unless a type of dark energy or another sort of exotic configuration is included. Many scientists hitherto have been captivated by this idea while others prefer to keep a skeptical position and opt for review part of the standard lore of cosmology, wondering if there

might be viable alternative theories of gravity which avoid unusual constituents. An active line of research towards this goal resides in braneworld cosmology where any proposed geometrical model should be self-consistent and able to reproduce an accurate cosmological evolution [1, 2].

In the brane scenario, the earlier model proposed by Regge and Teitelboim is an attempt to generalize GR [3], consisting of a brane Ricci scalar in addition to a brane cosmological constant Λ . This type of gravity is often referred to as *geodetic brane gravity* (GBG) [4] when no gravitational effect from the brane into the bulk is considered. In this stringy approach to classical gravitation, the embedding functions are alternative field variables instead of the four-dimensional metric components. GBG is parametrized by a conserved bulk energy, which renders a very interesting deviation from GR. Such energy is the source of a radiation-like energy characterizing higher dimensional cosmological models. Similarly, the Dvali-Gabadadze-Porrati (DGP) braneworld model has been considered as one of the most promising scenarios to study viable modifications of GR since, at low energies, it explains acceptably the late-time acceleration of our universe [5, 6, 7, 8]. Both brane models are conjectured to belong to an unified brane theory proposed in [9]. The main idea underlying this work is that these brane models may be closely connected by means of a geometric aggregate: a linearly extrinsic curvature term arising from the trajectory swept out by the brane. Indeed, this term introduces second-order derivative correction terms into the original GBG theory leading in turn to a robust geometrical effective model able to provide an accelerated branelike gravity resembling in certain limit that of the DGP approach.

If we are interested in maintaining the second order nature of the equations of motion to guarantee that no extra degrees of freedom propagate around the bulk, from a geometrical perspective, we have restrictions on the terms that we may include in any physically viable modification for GBG. Indeed, in addition to the first two Lovelock terms on the $(3 + 1)$ -dimensional worldvolume, namely the Λ and the \mathcal{R} scalars [10], we may only incorporate two possible boundary terms related to either a bulk Einstein-Hilbert term or a bulk Gauss-Bonnet term[‡]. These terms will necessarily introduce brane Lagrangians either proportional to K or to $K^3 - 3KK_{ab}K^{ab} + 2K^a_b K^b_c K^c_a$ as discussed in [11, 12, 13, 14, 15, 16] (see also below for notation). Now, as the original prescription for GBG does not consider the bulk gravity to be dynamical, we will focus on another $(3 + 1)$ -dimensional worldvolume geometrical term leading to second-order equations of motion. More specifically, in this work we consider only the linear term in the mean extrinsic curvature swept out by the brane as a small modification to the GBG. This fact provides an alternative mechanism to contrast the cosmological constant effects by introducing this peculiar kind of correction into the geodetic brane dynamics. Hence, in a FRW framework, we realize that the β parameter enforcing this brane correction contributes as either a catalyst or a preventer of the acceleration of this branelike universe in direct dependence of the sign of the mentioned parameter.

[‡] In the $(3 + 1)$ -dimensional case, the higher Lovelock terms are total derivatives and do not contribute to the equations of motion.

As a consequence of the invariance under reparametrizations of this model, the brane energy is conserved. This fact is important because it parametrizes the deviation of the Einstein limit as in GBG. We thereby obtain a useful expression to get a Friedmann-type equation, providing fingerprints of the cosmological evolution of this sort of universes. As a byproduct, when we consider no gravitational effects on the bulk from the brane, we reproduce the DGP cosmology accordingly where the role of the inverse crossover scale is played now by the β parameter. This would then suggest that in our approach, the regarded second order-correction term acts like the DGP bulk curvature effect thus modifying the acceleration behaviour of this branelike universe, and in addition, when we switch off the cosmological constant content, we are able to obtain an accelerated universe behaviour similar to the one developed in [17].

This extrinsic curvature correction term has drawn attention for a long time in several contexts. It was regarded in the study of hypersurfaces in differential geometry [18]. This correction term was also considered in the bending and shape determination of phospholipid membranes [19]. In the relativistic context, it appears in the improvement for the earlier attempt by Dirac to picture the electron as a bubble [20, 21], and more recently, it has been considered as an effective $4D$ field theory yielding one of the Galileon actions pursuing applications in particle physics and cosmology [12, 13, 22, 23, 24].

The paper is organized as follows. In Section 2, we deal with the geometrical aspects of the modified GBG. The variation casts out crucial results for the entire discussion of our approach. In Section 3, we specialize our model to a FRW geometry for the branelike universe. In Section 4, we provide a Friedman type equation, and also we analyse the effects that the K -term implements when the radiation-like energy is vanishing or is very small. Section 5 is dedicated to show how this model reproduces the DGP cosmology accordingly. In Section 6, we conclude and discuss our results.

2. The Geometrical Model

We consider a spacelike 3-brane Σ , propagating in a flat five-dimensional nondynamical Minkowski background spacetime with metric $\eta_{\mu\nu}$, ($\mu, \nu = 0, 1, \dots, 4$). To specify the brane trajectory, or worldvolume m , in the bulk, we set $x^\mu = X^\mu(\xi^a)$ to being the parametrization of the four-dimensional trajectory of Σ where x^μ are the local coordinates for the background spacetime, ξ^a are local coordinates for m , and X^μ the embedding functions ($a = 0, 1, 2, 3$). In general, the crucial derivatives of the parametrization are those encoded in the induced metric tensor $g_{ab} = \eta_{\mu\nu} e^\mu_a e^\nu_b = e_a \cdot e_b$ and the extrinsic curvature of m , $K_{ab} = -n \cdot D_a e_b$, where $D_a = e^\mu_a D_\mu$ and D_μ is the bulk covariant derivative and $e^\mu_a = \partial_a X^\mu$ stand for the tangent vectors to m . Moreover, n^μ denotes the spacelike unit normal vector to the worldvolume. It is defined implicitly by $n \cdot e_a = 0$ and we choose to normalize it as $n \cdot n = 1$.

We assume that the dynamics of Σ is described by the functional§

$$S[X] = \int_m d^4\xi \sqrt{-g} \left(\frac{\alpha}{2} \mathcal{R} + \beta K - \Lambda \right), \quad (1)$$

where the constants α and β have dimensions $[L]^{-2}$ and $[L]^{-3}$ in Planck units, respectively, and $g = \det(g_{ab})$, \mathcal{R} stands for the worldvolume Ricci scalar, $K = g^{ab}K_{ab}$ is the mean extrinsic curvature of m where g^{ab} denotes the inverse of g_{ab} . We have also included a cosmological constant term, Λ . The parameter β corresponds to a constant enforcing corrections to the GBG described by the original Regge-Teitelboim proposal which includes only a (3+1)-dimensional worldvolume Ricci scalar [3]. \mathcal{R} can be obtained either directly from the induced metric g_{ab} , or, in terms of the extrinsic curvature tensor via the contracted Gauss-Codazzi condition for immersed surfaces, $\mathcal{R} = K^2 - K_{ab}K^{ab}$. The local action (1) is invariant under reparametrizations of the worldvolume and its second-order derivative dependence on the fields should be noted. A key observation is that action (1), despite its dependence on second-order geometrical terms, leads to a second-order equation of motion (see below).

The response of action (1) to a deformation of the surface $X \rightarrow X + \delta X$ is characterized by a conserved stress tensor which can be straightforwardly computed from the knowledge of the Lagrangian $L = \frac{\alpha}{2}\mathcal{R} + \beta K - \Lambda$, [25]. We first have that $L^{ab} := \partial L / \partial K_{ab} = \alpha(Kg^{ab} - K^{ab}) + \beta g^{ab}$. Then the conserved stress tensor is given by

$$\begin{aligned} f^{a\mu} &= (Lg^{ab} - L^{ac}K^b_c) e^\mu_b + (\nabla_b L^{ab}) n^\mu \\ &= -(\alpha \mathcal{G}^{ab} + \beta S^{ab} + \Lambda g^{ab}) e^\mu_b, \end{aligned} \quad (2)$$

where $\mathcal{G}_{ab} = \mathcal{R}_{ab} - \frac{1}{2}\mathcal{R}g_{ab}$ is the worldvolume Einstein tensor with \mathcal{R}_{ab} being the corresponding Ricci tensor and $S_{ab} := K_{ab} - Kg_{ab}$. Furthermore, we have considered the Gauss-Codazzi condition $\mathcal{R}_{ab} = K K_{ab} - K_a^c K_{bc}$ and the fact that \mathcal{G}_{ab} and S_{ab} are conserved (see below). Note that this stress tensor is only tangent to m . In fact, this tensor captures relevant physical and geometrical information that is mediated by the worldvolume geometry. Moreover, (2) is identified as the Noether current associated with translation invariance of action (1). The classical brane trajectories can be obtained from the normal component of the covariant conservation law for (2), $n \cdot \nabla_a f^{a\mu} = 0$, where ∇_a is the worldvolume covariant derivative [25]. It is then straightforward to obtain a generalized geodetic-type equation governing the brane evolution

$$\mathcal{T}^{ab} K_{ab} = 0, \quad (3)$$

§ It is important to remark that we start from the four-dimensional action (1) in order to obtain the equation of motion where we assume a fixed background spacetime, i.e., there is no gravitational effect of the brane on the bulk which is the original premise in the GBG theory. In the general case when the bulk spacetime is dynamical, there are two possible contributions to the five-dimensional action coming from the intrinsic curvature terms of the two sides of the brane corresponding to the Gibbons-Hawking-York and Gauss-Bonnet terms. In this work, the K -term does not correspond to a pure Gibbons-Hawking-York term because we do not consider a five-dimensional Ricci scalar. It is worth mentioning that in the general case, it is possible to use the Dirac style brane variation to describe the Σ dynamics [9].

where

$$\mathcal{T}^{ab} = \alpha \mathcal{G}^{ab} + \beta S^{ab} + \Lambda g^{ab}. \quad (4)$$

This tensor is conserved and its conservation is supported by the Bianchi identity, $\nabla_a \mathcal{G}^{ab} = 0$, and the Codazzi-Mainardi condition for embedded surfaces, $\nabla_a S^{ab} = 0$. The equation of motion (3) is of second order in derivatives of the embedding functions because of the presence of the extrinsic curvature tensor. This is so even though we have the presence of second-order derivative terms in our model through the scalars \mathcal{R} and K . Additionally, we can construct the physical quantity given by

$$\tilde{\pi}_\mu = \sqrt{h} \eta_a f_\mu{}^a = -\sqrt{h} \mathcal{T}^{ab} \eta_a e_{\mu b}, \quad (5)$$

where η^a denotes the timelike unit normal vector to Σ when it is viewed into m [26], and $h := \det(h_{AB})$ with $h_{AB} = g_{ab} \epsilon^a{}_A \epsilon^b{}_B$ being the spatial metric on Σ and $\epsilon^a{}_A$ are the tangent vectors to Σ ($A, B = 1, 2, 3$). In other words, when Σ is considered as a spacelike surface in the worldvolume m described by the embedding $\xi^a = \chi^a(u^A)$, with χ^a being the corresponding embedding functions, we can obtain an orthonormal basis defined at each point of Σ given by $\{\epsilon^a{}_A, \eta^a\}$, where η^a satisfies $g_{ab} \eta^a \eta^b = -1$ (see Ref. [27] for more details). On physical grounds, expression (5) corresponds to the conserved linear momentum density associated with the brane Σ .

3. Modified Geodetic Brane Cosmology

A simple but interesting brane geometry is provided by a spherical configuration. Suppose that Σ evolves in a five-dimensional Minkowski spacetime, $ds_5^2 = -dt^2 + da^2 + a^2 d\Omega_3^2$, where $d\Omega_3^2 = d\chi^2 + \sin^2 \chi d\theta^2 + \sin^2 \chi \sin^2 \theta d\phi^2$ denotes the metric of the unit 3-sphere. Assuming the universe to be homogeneous, isotropic and closed, it leads to consider

$$x^\mu = X^\mu(\xi^a) = (t(\tau), a(\tau), \chi, \theta, \phi), \quad (6)$$

to be a parametric representation of the worldvolume m , where τ is the proper time for an observer at rest with respect to the brane. This is the geometry of the standard Friedmann-Robertson-Walker (FRW) case.

The orthonormal basis adapted to m is given by the four tangent vectors $e^\mu{}_a$ complemented with the unit spacelike normal vector

$$n^\mu = \frac{1}{N}(\dot{a}, \dot{t}, 0, 0, 0), \quad (7)$$

where we have introduced the function $N = \sqrt{\dot{t}^2 - \dot{a}^2}$. The overdot denotes differentiation with respect to τ . The metric induced from the background spacetime is

$$ds_4^2 = -N^2 d\tau^2 + a^2 d\Omega_3^2 = g_{ab} d\xi^a d\xi^b. \quad (8)$$

We can identify immediately the spatial metric on Σ and in consequence its determinant, $h = a^6 \sin^4 \chi \sin^2 \theta$. The coordinate system defined by (8) and (7) promotes the following

non-vanishing components of the extrinsic curvature

$$K^\tau{}_\tau = \frac{\dot{t}^2}{N^3} \frac{d}{d\tau} \left(\frac{\dot{a}}{\dot{t}} \right), \quad (9a)$$

$$K^\chi{}_\chi = K^\theta{}_\theta = K^\phi{}_\phi = \frac{\dot{t}}{Na}. \quad (9b)$$

The Ricci scalar associated to the metric (8) and the mean extrinsic curvature are computed accordingly,

$$\mathcal{R} = \frac{6\dot{t}}{N^4 a^2} (a\ddot{a}\dot{t} - a\dot{a}\ddot{t} + N^2 \dot{t}), \quad (10)$$

$$K = \frac{1}{N^3} (\dot{t}\ddot{a} - \dot{a}\ddot{t}) + \frac{3\dot{t}}{aN}. \quad (11)$$

Note the linear dependence in the accelerations that these geometrical scalars possess. In addition, associated with this geometry, we have the nonvanishing components of the worldvolume Einstein tensor,

$$\mathcal{G}^\tau{}_\tau = -\frac{3\dot{t}^2}{N^2 a^2}, \quad (12)$$

$$\mathcal{G}^\chi{}_\chi = \mathcal{G}^\theta{}_\theta = \mathcal{G}^\phi{}_\phi = -\frac{\dot{t}^2}{N^4 a^2} \left[2a\dot{t} \frac{d}{d\tau} \left(\frac{\dot{a}}{\dot{t}} \right) + N^2 \right], \quad (13)$$

and

$$S^\tau{}_\tau = -\frac{3\dot{t}}{Na}, \quad (14)$$

$$S^\chi{}_\chi = S^\theta{}_\theta = S^\phi{}_\phi = -\frac{\dot{t}}{N^3 a} \left[a\dot{t} \frac{d}{d\tau} \left(\frac{\dot{a}}{\dot{t}} \right) + 2N^2 \right]. \quad (15)$$

The mechanical law governing the evolution of the brane Σ is obtained from equation (3). After a lengthy but straightforward computation, it yields the equation of motion

$$\frac{d}{d\tau} \left(\frac{\dot{a}}{\dot{t}} \right) = -\frac{N^2}{a^2 \left(\frac{\dot{t}}{a} \right)} \frac{\left(\frac{\dot{t}^2}{a^2} - 3\bar{\Lambda}N^2 + 6\bar{\beta}N\frac{\dot{t}}{a} \right)}{\left(3\frac{\dot{t}^2}{a^2} - \bar{\Lambda}N^2 + 6\bar{\beta}N\frac{\dot{t}}{a} \right)}, \quad (16)$$

which involves second derivatives of the field variables a and t . Hereafter, a bar over a letter denotes the quotient by 3α , e.g. $\bar{\Lambda} = \Lambda/3\alpha$, etc. The reparametrization invariance of the model (1) dictates that for every solution for the expansion rate $a(\tau)$, we have a gauge freedom to choose in connection with a function for $t(\tau)$. This fact will be used repeatedly in the subsequent developments.

3.1. Inclusion of matter

As expected, action (1) is permitted to depend on additional fields, like matter sources. Their dynamical contributions are obtained through the stress tensor $T_{ab} := (-2/\sqrt{-g})\delta S_m/\delta g^{ab}$ where S_m denotes a matter action. The form of the equation of motion remains unchanged when we add T_m^{ab} to the original one in equation (3) [4],

by modifying the tensor \mathcal{T}^{ab} as follows: $\mathcal{T}^{ab} \mapsto \mathcal{T}^{ab} - T_m^{ab}$. According to this line of reasoning, the conserved stress tensor (2) gets a similar contribution from the matter Lagrangian.

Now, a good acceptance for the energy-momentum tensor that is compatible with the assumed homogeneity and isotropy of the universe is given by

$$T_m^{ab} = (\rho + P)\eta^a\eta^b + P g^{ab}, \quad (17)$$

where $\rho = \rho(a)$ is the energy density of the fluid and $P = P(a)$ its pressure. Now, by considering (12)-(15) and the matter field content enclosed in (17), a straightforward computation from the expression (5) for $\tilde{\pi}_\mu$, it yields the expressions for the momenta conjugate to $\{t, a\}$

$$\pi_t = \frac{at}{N^3} [\dot{a}^2 + N^2 (1 - a^2 \bar{\Lambda} - a^2 \bar{\rho}) + 3\bar{\beta} N a \dot{t}], \quad (18)$$

$$\pi_a = -\frac{a\dot{a}}{N^3} [\dot{a}^2 + N^2 (1 - a^2 \bar{\Lambda} - a^2 \bar{\rho}) + 3\bar{\beta} N a \dot{t}], \quad (19)$$

respectively, where we have absorbed a global constant in (18) and (19) in order to avoid the density character of the momenta. Although we have so far restricted ourselves to the spherical case for simplicity, it may be convenient to generalize these expressions in order to include also the cases of hyperbolic and flat geometries. To this end, we can choose alternative parametrizations for equation (6) (see for example [28] for a detailed variety of embeddings) which requires analogue developments. At this stage, we mention only that depending on the parametrization chosen, the momenta π_t and π_a acquire a different form but, however, they pleasantly lead to an unaltered form for the energy conservation law^{||} (see (20) below). Hereafter, the three possible geometries for the universe will be considered in the successive equations through the parameter k , ($k = -1, 0, 1$) and the use of the cosmic gauge, $N = 1$.

4. Friedmann Type Equation

The first integral of equation (16) corresponds to the Friedmann equation associated to our model (1). To find out, we proceed by considering the conserved physical quantities arising in our model. The reparametrization invariance of model (1), in the time coordinate t , dictates that equation (18) is conserved. For a general geometry of the universe and by considering the cosmic gauge, equation (18) is promoted to

$$-\frac{E}{a^4} := \frac{(\dot{a}^2 + k)^{1/2}}{a} \left[\frac{(\dot{a}^2 + k)}{a^2} - (\bar{\Lambda} + \bar{\rho}) \right] + 3\bar{\beta} \frac{(\dot{a}^2 + k)}{a^2}, \quad (20)$$

^{||} For illustration, following [28] for the open Universe case, the quantities $K^\tau{}_\tau$, $\mathcal{G}^\tau{}_\tau$ and $S^\tau{}_\tau$ remain unaltered as (9a), (12) and (14) so the term $T^\tau{}_\tau$ with an associated lapse function given by $\mathcal{N} = \sqrt{\dot{a}^2 - \dot{\tau}^2}$. In such a case the only change in form lies in the quantities $\mathcal{G}^\chi{}_\chi = \mathcal{G}^\theta{}_\theta = \mathcal{G}^\phi{}_\phi$ given by $\mathcal{G}^\chi{}_\chi = -\frac{\dot{a}^2}{a^2 \mathcal{N}^4} \left[2a\dot{\tau} \frac{d}{d\tau} \left(\frac{\dot{a}}{\dot{\tau}} \right) + \mathcal{N}^2 \right]$ and in consequence in $T^\chi{}_\chi$. Therefore, by repeating the development above we obtain a momentum conjugate to \mathcal{T} given by $\pi_y = \frac{a^3 \dot{\tau}}{\mathcal{N}} \left[\frac{\dot{\tau}^2}{a^2 \mathcal{N}^2} - (\bar{\Lambda} + \bar{\rho}) + \frac{3\bar{\beta} \dot{\tau}}{a \mathcal{N}} \right]$ which leads to an energy conservation law given by (20) with $k = -1$.

where $E := -\pi_t$ is the conserved brane energy and it parametrizes the deviation from the Einstein limit in the sense that as $E \rightarrow 0$ and $\beta \rightarrow 0$ together, the Einstein cosmology is recovered. In terms of this energy, the equation of motion (3) in the presence of a matter configuration, results

$$\frac{\ddot{a}}{a} = \frac{\left(\frac{\dot{t}^2}{a^2}\right) \left[\frac{E}{a^4} - 3\bar{\beta}\frac{\dot{t}^2}{a^2} + 2(\bar{\Lambda} + \bar{\rho})\frac{\dot{t}}{a} - 3(\bar{\rho} + \bar{P})\frac{\dot{t}}{a}\right]}{\left[-\frac{3E}{a^4} - 3\bar{\beta}\frac{\dot{t}^2}{a^2} + 2(\bar{\Lambda} + \bar{\rho})\frac{\dot{t}}{a}\right]}. \quad (21)$$

Now, by defining $\mathcal{X} := \dot{t}/[a(\bar{\Lambda} + \bar{\rho})^{1/2}] = (\dot{a}^2 + k)^{1/2}/[a(\bar{\Lambda} + \bar{\rho})^{1/2}]$ we are able to rewrite the energy equation (20) as $\mathcal{X}^3 + 3\beta^*\mathcal{X}^2 - \mathcal{X} + 2E^*a^{-4} = 0$ where we have introduced the notation, $\beta^*(a) = \bar{\beta}/(\bar{\Lambda} + \bar{\rho})^{1/2} = \beta/[3\alpha(\Lambda + \rho)]^{1/2}$ and $E^*(a) = E/[2(\bar{\Lambda} + \bar{\rho})^{3/2}] = (E/2)[3\alpha/(\Lambda + \rho)]^{3/2}$. Now, by considering the usual Liouville change of variable $\mathcal{X} := \mathcal{Y} - \beta^*$, the cubic equation for \mathcal{X} transforms into

$$\mathcal{Y}^3 - (1 + 3\beta^{*2})\mathcal{Y} + \left[\beta^*(1 + 2\beta^{*2}) + \frac{2E^*}{a^4}\right] = 0, \quad (22)$$

which is an incomplete cubic equation with real coefficients. Taking into account the identities $4\cos^3\theta - 3\cos\theta = \cos 3\theta$ and $4\cosh^3\theta - 3\cosh\theta = \cosh 3\theta$ and the fact that $(1 + 3\beta^{*2}) > 0$, the physical solution for equation (22) is given by

$$\mathcal{Y} = 2\sqrt{\beta^{*2} + \frac{1}{3}} F \left\{ \frac{1}{3} F^{-1} \left[\frac{\beta^*(\beta^{*2} + \frac{1}{2}) + \frac{E^*}{a^4}}{(\beta^{*2} + \frac{1}{3})^{3/2}} \right] \right\}, \quad (23)$$

where

$$F(x) = \begin{cases} \cosh x, & \text{for } |x| > 1, \\ \cos x, & \text{for } |x| \leq 1. \end{cases} \quad (24)$$

By considering dust, $\rho = \rho_{m0}/a^3$, and inserting the expression (23) into the definition for \mathcal{X} , we obtain

$$\frac{(\dot{a}^2 + k)^{1/2}}{a(\bar{\Lambda} + \bar{\rho}_{m0}a^{-3})^{1/2}} + \beta^* = 2\sqrt{\beta^{*2} + \frac{1}{3}} F \left\{ \frac{1}{3} F^{-1} \left[\frac{\beta^*(\beta^{*2} + \frac{1}{2}) + \frac{E^*}{a^4}}{(\beta^{*2} + \frac{1}{3})^{3/2}} \right] \right\}, \quad (25)$$

where $\bar{\rho}_{m0} = \rho_{m0}/3\alpha$. Then, when we square this equation followed by a rearrangement, we obtain the Friedmann type equation in the fashion

$$\dot{a}^2 + U(a, E) = 0, \quad (26)$$

where we can identify an effective potential parametrized by the constant E through the parameter Ω_{dr} , as defined below,

$$\begin{aligned} \frac{U(a, E)}{H_0^2} = & -\Omega_{k,0} - \frac{a^2}{9} \left\{ 2 \left[\Omega_{\beta,0}^2 + 3 \left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{a^3} \right) \right]^{1/2} \times \right. \\ & \times F \left[\frac{1}{3} F^{-1} \left(\frac{\Omega_{\beta,0} \left[\Omega_{\beta,0}^2 + \frac{9}{2} \left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{a^3} \right) \right] - \frac{27\Omega_{dr}}{2a^4}}{\left[\Omega_{\beta,0}^2 + 3 \left(\Omega_{\Lambda,0} + \frac{\Omega_{m,0}}{a^3} \right) \right]^{3/2}} \right) \right] - \Omega_{\beta,0} \left. \right\}^2. \end{aligned} \quad (27)$$

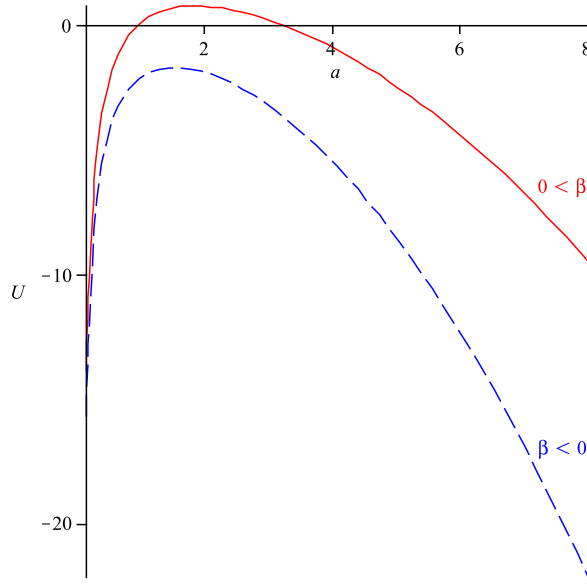


Figure 1. The effective potential U describing possible brane trajectories for a closed universe and some parameter choices [$\Omega_{dr} = 0.1, \Omega_{\Lambda,0} = 0.2, \Omega_{m,0} = 2.5(\Omega_{\beta,0} = 0.5, \Omega_{k,0} = -2.59), (\Omega_{\beta,0} = -0.5, \Omega_{k,0} = -0.93)$]. The solid curve describes the potential involving positive β . The dashed curve denotes the potential involving negative β .

Here we have introduced the energy density parameters defined by $\Omega_{k,0} := -k/H_0^2$, $\Omega_{\Lambda,0} := \Lambda/(3\alpha H_0^2)$, $\Omega_{m,0} := \rho_{m,0}/(3\alpha H_0^2)$, $\Omega_{\beta,0} := \beta/(\alpha H_0)$ and $\Omega_{dr} := -E/H_0^3$, the latter being the so-called *dark radiation-like energy* density parameter. As customary, H_0 is the Hubble constant. In what follows, we develop a standard mechanical analysis for the motion of a single non-relativistic particle moving with zero energy in the potential $U(a, E)$.

In Figure (4), we depict this potential for an energy density matter configuration in the brane of the form $\rho \propto a^{-3}$ where we have considered some parameter choices. We infer that $U(a, E)$ becomes singular at $a \rightarrow 0$ and when $a \rightarrow \infty$ then $U(a, E) \rightarrow -\infty$. In addition, for some of the chosen parameters, the universe will expand forever at an ever increasing rate (Big Chill) because the dashed curve does not intersect the horizontal axis and this implies that there is no returning point with $\dot{a} = 0$. For some other parameters, we observe a Big Bounce behaviour because in this case the potential intersects the horizontal axis and there is a returning point¶. These brane trajectories for the universe are strongly in dependence of the conserved energy values. Thus, the message is clear. Negative values of the parameter β , $\beta < 0$, cause the universe to accelerate whereas positive values, $\beta > 0$, cause it to decelerate, i.e. the β parameter works as a preventer or a catalyst of the acceleration of this type of universe.

¶ We follow the terminology employed in [31] to refer the term Big Chill for a universe that expands forever and Big Bounce for a universe with no Big Bang singularity.

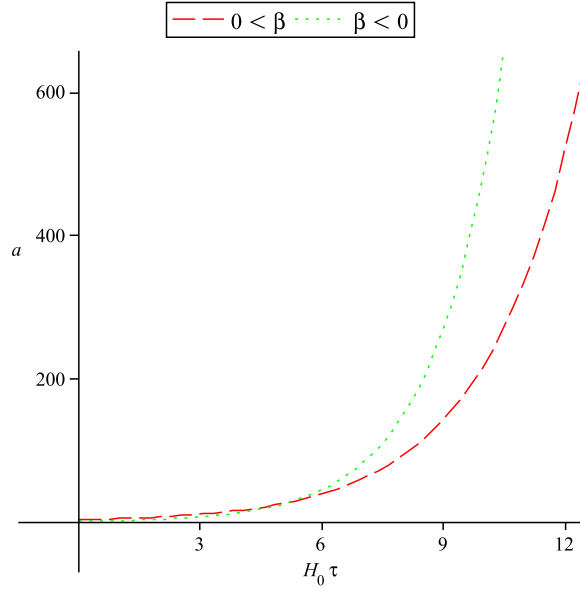


Figure 2. The scale factor a as a function of $H_0\tau$ for a closed universe with the parameter choices $[\Omega_{dr} = 0.1, \Omega_{\Lambda,0} = 0.2, \Omega_{m,0} = 2.5, (\Omega_{\beta,0} = 0.5, \Omega_{k,0} = -2.59), (\Omega_{\beta,0} = -0.5, \Omega_{k,0} = -0.93)]$. The dashed curve denotes the potential involving positive β parameter and the dotted curve involves negative β parameter.

Moreover, from equation (26), it is possible to plot the scale factor a versus $H_0\tau$ for some parameter choices in the case of a closed universe, see figure (4). Note that, in dependence on the sign of β , the universe would have to start off in a very special form. The scale factor behaviour depends sensitively on what the initial value is.

Another conveniently form to rewrite the Friedmann equation (26) is in terms of the Hubble parameter $H := \dot{a}/a$ and the energy density parameters associated to the model. Thus, equation (26) reads

$$\left(\frac{H^2}{H_0^2} - \frac{\Omega_{k,0}}{a^2}\right)^{1/2} \left(\frac{H^2}{H_0^2} - \frac{\Omega_{k,0}}{a^2} - \frac{\Omega_{m,0}}{a^3} - \Omega_{\Lambda,0}\right) + \Omega_{\beta,0} \left(\frac{H^2}{H_0^2} - \frac{\Omega_{k,0}}{a^2}\right) = \frac{\Omega_{dr}}{a^4}. \quad (28)$$

Clearly, one may verify that the normalization condition is obtained straightforwardly from (28) by evaluating at the present moment, yielding

$$(1 - \Omega_{k,0})^{1/2}(1 - \Omega_{k,0} - \Omega_{m,0} - \Omega_{\Lambda,0}) + \Omega_{\beta,0}(1 - \Omega_{k,0}) = \Omega_{dr}. \quad (29)$$

On the other hand, for consistency when $\Omega_{dr} = \Omega_{\beta,0} = 0$, it is evident that the standard cosmology is fully recovered [1, 32].

The general behaviour of the scale factor can be obtained from the normalization condition and the effective potential. In order to extract physical information, we have to find the roots of the effective potential because this gives us the possible existence of returning points for the scale factor. If there are no returning points, we have an universe that expands forever. The other possible situation, in our case, involves two returning points corresponding to a universe without initial singularity. The procedure is as follows. From equation (29) we have a cubic equation for $\Omega_{k,0}$. Once we choose a

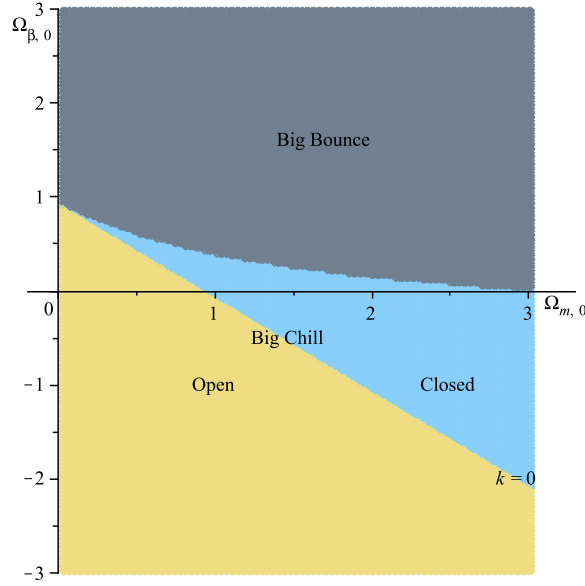


Figure 3. The types of expansion for the universe containing both matter and β content for the parameter choices $\Omega_{dr} = 0.1$ and $\Omega_{\Lambda,0} = 0.2$.

root for $\Omega_{k,0}$ we use equation (28) with $H = 0$ and obtain a cubic equation for the scale factor which is equivalent to find the roots of the effective potential. The number of roots gives us the overall behaviour of the universe in terms of an universe that expands forever or without Big Bang.

We illustrate the evolution of the scale factor for one of the roots of $\Omega_{k,0}$ in figure (4). It is possible to extend the parameter ranges for $\Omega_{k,0}$ and $\Omega_{\beta,0}$ but we do not find any new physical information. The other root presents a similar general evolution of the universe including Big Bounce and Big Chill regions. The remaining root has only positive values in the parameters region of figure (4) corresponding to hyperbolic geometry.

5. DGP like behaviour from modified GBG cosmology

As is well known, the radiation-like contribution in (28) is an effect emerging from the bulk. A very interesting physical approximation of our model is when we have no contribution of dark radiation-like energy. In this case, the condition $\Omega_{dr} = 0$ brings, in the cosmic gauge, the generalized Friedmann equation (20) into the form

$$H^2 + \frac{k}{a^2} + 3\bar{\beta}\sqrt{H^2 + \frac{k}{a^2}} = \bar{\Lambda} + \bar{\rho}. \quad (30)$$

This equation stands as the analogue of the expression that emerges for the normal (non-self-accelerating) branch of the DGP theory when $\beta > 0$. The case $\beta < 0$ corresponds to the self-accelerating branch in the DGP approach which in general is ruled out of the brane world scenario due to the appearance of a number of pathologies [7, 8]. Note

also that, when we re-organize equation (30) as in the standard relativistic form, we can identify an effective cosmological constant given by

$$\Lambda_{eff} := \Lambda - 3\beta\sqrt{H^2 + \frac{k}{a^2}}. \quad (31)$$

Note that the β parameter plays the role of the inverse of the crossover scale r_c , responsible of the transition from $4D$ to $5D$ behaviour in the degravitation property of the DGP approach [1, 6]. In fact, β is related to the $5D$ Planck mass as one can observe from the set of purely gravitational actions that give rise to the Galileon field theory of brane cosmology [11]. Hence, in our approach, the responsible of the (non-) self-acceleration of the Universe is the parameter β characterizing the intrinsic properties of the geometrical surface. In addition, for an arbitrary β , from equation (29) we can obtain the corresponding normalization condition

$$\sqrt{1 - \Omega_{k,0}} = -\frac{\Omega_{\beta,0}}{2} + \sqrt{\left(\frac{\Omega_{\beta,0}}{2}\right)^2 + \Omega_{m,0} + \Omega_{\Lambda,0}} \quad (32)$$

which can be recognized as the normalization condition in the DGP approach by defining $\Omega_{r_c} = (\Omega_{\beta,0}/2)^2 = \beta^2/(2\alpha H_0)^2$ [1, 17].

The Friedmann equation acquires a non conventional form when we consider a nonvanishing radiation-like contribution. For example, in the reasonable case of $|\Omega_{dr}| \ll 1$ [33], and $\Lambda = 0$, after a lengthy but straightforward computation, from equation (28) the Friedmann equation reads

$$\frac{H^2}{H_0^2} - \frac{\Omega_{k,0}}{a^2} = \left[-\frac{\Omega_{\beta,0}}{2} + \sqrt{\left(\frac{\Omega_{\beta,0}}{2}\right)^2 + \Omega_{m,0}} \right]^2 + f(a, \Omega_{\beta,0}, \Omega_{m,0}, \Omega_{dr}), \quad (33)$$

where f is a quite complicated function which explicitly reads

$$f = \frac{3 \left[-\frac{\Omega_{\beta,0}}{2} + \sqrt{\left(\frac{\Omega_{\beta,0}}{2}\right)^2 + \Omega_{m,0}} \right] \left(D_+^{1/3} - D_-^{1/3} \right) \Omega_{dr}}{a^4 \sqrt{\left[\Omega_{\beta,0} \left(\Omega_{\beta,0}^2 + \frac{9}{2} \frac{\Omega_{m,0}}{a^3} \right) \right]^2 - \left(\Omega_{\beta,0}^2 + \frac{3\Omega_{m,0}}{a^3} \right)^3}}, \quad (34)$$

and $D_{\pm} = -\Omega_{\beta,0} \left(\Omega_{\beta,0}^2 + \frac{9}{2} \frac{\Omega_{m,0}}{a^3} \right) \pm \sqrt{\left[\Omega_{\beta,0} \left(\Omega_{\beta,0}^2 + \frac{9}{2} \frac{\Omega_{m,0}}{a^3} \right) \right]^2 - \left(\Omega_{\beta,0}^2 + \frac{3\Omega_{m,0}}{a^3} \right)^3}$. Clearly, expression (33) specializes to the Friedmann equation found in [17] for the analysis of an accelerated universe without cosmological constant. In addition, from equations (33) and (34), two features should be emphasized. At large distances, $a \rightarrow \infty$ gives rise to $f \rightarrow 0$ and we have a slight modification to the DGP accelerated universe behaviour. On the other hand, however, when $a \rightarrow 0$, the function f increases considerably and a significant effect is expected which will strongly modify the quantum cosmology approach for our model. Therefore, we find that when Ω_{dr} fades away then modified GBG leads to a late-time self-acceleration and in consequence we have an interesting alternative effective model supporting this degravitation characteristic.

6. Concluding remarks

In this paper, we have shown that GBG modified by a curvature brane scalar defined by a linear extrinsic curvature term leads us to reproduce under certain conditions a similar brane cosmological behaviour as in the DGP setup. Contrarily to the DGP approach, we considered an empty fixed bulk which, however, leads us to reproduce similar dynamical equations. In addition, we showed that the effective potential emerging in our model exhibits an accelerated behaviour for this universe as DGP theory does. In this modified GBG, the inverse DGP crossover scale r_c is played by the parameter related to the K -term. We thus observe that this model mimics the dynamics of DGP approach with the important exception that in our case such effects directly result from the geometrical characteristics of the brane through the parameter β . Even though gravity was switched off from the beginning in our setup all this happens so because modified GBG is enclosed in the set of purely gravitational $4D$ brane theories that gives rise to the Galileon field theory where that peculiar type of scalar fields is able to create accelerating spacetimes similar to those of the DGP setup [11, 35], which are argued to explain also dark energy. From another point of view, we study part of the Galileon field theory, restricted to the first three Lovelock brane Lagrangians directly.

This prescription surely constitutes an interesting theoretical alternative to analyse certain subjects in braneworld cosmology. In particular, we confirm that the acceleration description of brane universes can be formulated differently from what we have been accustomed to so far. Then, motivated by the equivalence at the effective level, with DGP gravity, we believe that this modified GBG deserves a deep exploration at the classical canonical level, followed by an extensive study of its quantum mechanical aspects in order to discern on the physical viability of the brane model. Our work will not stop here. We are interested in understanding how much an Ostrogradski-Hamiltonian development of model (1) is sensitive to the fact that the ghost and tachyon states appear in this scenario [21, 34]. Alternatively, we believe that our approach opens the scope of brane theories to be included as a correction terms, like the Gauss-Bonnet counter-term, yielding also second order equations of motion [29, 30] improving thus the GBG framework. It is worth saying that the terms appearing in the present model belong to a wide range of effective field brane models related to the so-called Galileons which possess interesting symmetry properties that also deserve an exhaustive investigation in view of their potential applications to particle physics and cosmology [11, 12, 13, 22, 23, 24]. The results of these points will be presented elsewhere.

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